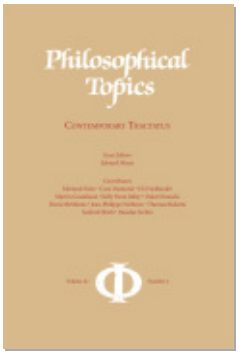
Analysis, Independence, Simplicity, and the General Sentence-Form



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Analysis, Independence, Simplicity, and the General Sentence-Form

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ABSTRACT. The first section of the paper argues that, in the context of Wittgenstein's intentional understanding of the truth-functional construction of sentences, the independence of elementary sentences is required for every application of a truth-operation to have the same significance. The second section of the paper presents a 'top-down' interpretation of Tractarian analysis. There is no characterization of the bottom level of analysis apart from the general sentence-form; the only constraint on analysis is that apparently manifest logical relationships among colloquial sentences with sense be represented or otherwise accommodated using only the logical resources built into the general sentence-form. The third section of the paper considers how those logical resources might be applied in Tractarian analysis. Central here is the use of form-series to construct infinitary truth-functions. The section develops a view of Tractarian complexes which accommodates some colloquially manifest logical relations as presuppositions that are made explicit in analysis. Finally, I note that Tractarian analysis, as presented in this paper, is immune to the 'color exclusion' objection as formulated in Wittgenstein's 1929 paper "Some Remarks on Logical Form."

At the core of the *Tractatus* is the general sentence-form (allgemeine Satzform). With great fanfare, 4.5 concludes: "The general sentence-form is: That's the way it

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is [Es verhält sich so und so]." Previously, 4.4 states: "A sentence is an expression of agreement and disagreement with the truth-possibilities of elementary sentences." 5 restates this idea; 6 formally presents it as a sentence-variable. The centrality of the general sentence-form to the philosophy of the *Tractatus* is underscored by the mention of it in *Philosophical Investigations* §65:

> Someone might object against me: "You make things easy for yourself! You talk about all sorts of language-games, but have nowhere said what is essential to a language-game, and so to language . . . . So you let yourself off the very part of the investigation that gave you the most headache, the part concerning the *general form of the sentence* and of language."

Surely, the general sentence-form is, in some sense, the summary and culmination of the *Tractatus* account of representation. However, as Elizabeth Anscombe warns at the beginning of her estimable *Introduction to Wittgenstein's Tractatus,* it is all too easy to take the *Tractatus* conception of representation to be

> a merely external combination of two theories: a 'picture theory' of elementary propositions (viz. that they have meaning by being 'logical pictures' of elementary states of affairs), and the theory of truth-functions as an account of non-elementary sentences.[1]

The effort to view things otherwise confronts pressing questions. What motivation is there for the independence of elementary sentences? Why is the logically significant construction of sentences exhausted by truth-functional construction and what are the means for truth-functional construction? What is the general character of the names that figure in elementary sentences and the simple objects they mean? What is it for the sentences with sense of colloquial language all to be truth-functions of independent elementary sentences?

The first section of this essay begins with the first question and argues that the understanding of truth-functions built into the conception of sentences as logical pictures motivates the independence of elementary sentences. This answer to the first question brings with it a motivation for identifying sentences generally with truth-functions of elementary sentences. The analysis of sentences with sense is the task of rewriting them to make perspicuous what truth-function of which elementary sentences each is. The second section of the paper maintains the general sentence-form on its own is the sole and sufficient constraint on analysis. This role for analysis reveals the kind of grasp of elementary sentences the *Tractatus* takes us to have, and illuminates the colorless character of Tractarian simple objects. It also suggests a general characterization of the forms of elementary sentences and their component names. The abstract characterization of Tractarian analysis constrained only by the general sentence-form gives no idea how analysis might capture the logical relations manifest within colloquial language in Tractarian terms. The third section of the paper illustrates two strategies for applying the considerable logical resources of the general sentence-form in analysis. Reflection on these examples suggests that it was not unreasonable for Wittgenstein to think that the

logical resources built into the general sentence-form were adequate for Tractarian analysis. This conclusion in turn raises questions as to why Wittgenstein, when he returned to philosophical work in 1929, rejected the *Tractatus*.

I

There are two distinctive features to the characterization of sentences offered by the general sentence-form. The first is the limitation of the "logically significant" (5.233) construction of sentences to truth-functional construction. Implicit in this limitation is a class of sentences that are not constructed from other sentences, Wittgenstein's elementary sentences.[2] The second is the independence of elementary sentences. No reason or motivation is explicitly provided for these features. The independence of elementary sentences and the states of things they assert appears particularly arbitrary. To begin, I shall argue that the independence of elementary sentences is grounded in Wittgenstein's conception of the truth-functional construction of sentences. This will lead to the sufficiency of truth-functional construction.

We conceive of truth-functional combinations of sentences extensionally: the truth-functional connectives are characterized by the way in which the truth-value of the sentences they yield depends on the truth-values of the sentences they join. In contrast, Wittgenstein's understanding is intentional.[3] It exploits the modal aspect of his notion of truth, and presupposes the independence of elementary sentences. The Tractarian conception of truth-functions is then neither familiar nor straightforward. I will first simply present the Tractarian account of truth-functions. Then I will motivate the two distinctive features of the *Tractatus* characterization of sentences by considering a salient alternative account of truth-functional construction that does not require independent elementary sentences.

The *Tractatus* conception of the truth-functional construction of sentences is rooted in the conception of sentences as pictures. The basic idea behind picturing is that a picture or model represents reality by having something in common with it. In a model, items are arranged in a particular way. These picturing items go proxy for things. That the picturing items are arranged in a particular way in the model represents that the things for which the items go proxy are likewise arranged in reality. The picture is then true, if the things are in reality as their proxies are in the picture. Picturing thus requires a sort of identity between the ways that the picturing elements can be related to form a picture and the ways that the things can be related in fact. 2.17 calls this isomorphism "pictorial form" (Form der Abbildung).

For present purposes, what is important here is how this conception of representation as picturing introduces a modal element into the notion of truth and its material mode reflection, the notion of fact. Pictorial form is constituted by shared possibilities. Hence, the truth of pictures is contingent truth. Any correct

picture might have been false; anything that is the case might not have been the case. Nevertheless, as far as we have gone, nothing about pictorial form requires the truth or falsity of one picture to be independent of others with the same pictorial form.[4]

What the 2.18s add to this conception of picturing is central to what follows. 2.18 tells us that any picture of reality, whatever its pictorial form, must share logical form with reality: "Every picture is *also* a logical picture" (2.182). To be a logical picture is to represent a possible situation in logical space, and this possible situation is its sense (2.202; 2.221; 4.031b). Each represented situation is in itself a possibility. However, the obtaining of one situation may guarantee the obtaining of another or vice versa; it may guarantee the non-obtaining of the other situation or vice versa; or finally the obtaining of either situation may be independent of the obtaining of the other. Each possible situation is logically related to every other in these ways, and these logical relationships among them are intrinsic to the identity of the possibilities. In the logical relationships among possible situations we find the necessity that gives the possibility built into the notions of truth and fact its significance. 6.37 says, "There is only *logical* necessity." This is not a conclusion: it is built into the *Tractatus* conception of logical pictures.

As a result of the possibilities of truth and falsity intrinsic to logical pictures of reality, from representations of possible situations we can form representations of further, logically related possible situations. In particular, from a representation that a particular situation holds we can form a representation that the situation does not hold. Furthermore, a representation that possibility A holds and one that possibility B holds can be combined into one representation that either the one possibility or the other holds. Again, this is an aspect of the metaphor of logical space. Think of possible situations as locations in logical space, and sentences as specifications of these locations by means of coordinates. (See the 3.4s.) We can, from the specification of locations in logical space, form representations of the unions, intersections, and complements of those locations. Here then we have our first conclusion: truth-functional combinability is intrinsic to logical pictures.

Truth-functions are introduced in connection with elementary sentences in the 4.3s and 4.4s. 4.4 says that sentences generally are expressions of agreement and disagreement with the truth-possibilities of elementary sentences. (See 4.4 and 5.) So, the disjunction of two elementary sentences agrees with the three truth-possibilities in which either or both of the elementary sentences are true and disagrees with the truth-possibility that both are false. 4.31 introduces the familiar tabular notation for enumerating the truth-possibilities of a list of elementary sentences. 4.442 uses these enumerations in sentence-signs expressing truth-functions in the following way: to the right of each row of Ts and Fs indicating a truth-possibility for elementary sentences, write a T to express agreement with that truth-possibility, and write nothing or write an F to indicate disagreement. If we fix a rule for systematically enumerating the truth-possibilities, we can abbreviate the tabular sentence-sign by writing the final column of the tabular sign as a

row of Ts and Fs before the elementary sentences combined. So the disjunction of two elementary sentences "p" and "q" might be expressed by

(TTT —)(p,q).

Following the usage of the *Tractatus,* I have used the word "truth-function" to speak of sentences. Corresponding to our truth-functional connectors, the *Tractatus* introduces truth-operations (5.2341b). An operation is a way of constructing sentences from sentences in a logically significant way (5.23; 5.233). The sentences to which the operation is applied are the bases for that application of the operation. 5.3 characterizes truth-operations as "the way in which a truth-function arises from elementary sentences." The passage continues:

> It is of the essence of a truth-operation that in the same way that a truth-function is produced from elementary sentences, from truth-functions a new truth-function is produced.[5]

How are we to understand this talk of *a way* of producing a truth-function? 5.31 states that the tabular notation introduced in connection with elementary sentences at 4.31 has "a meaning (Bedeutung)," if 'p', 'q', and 'r', etc., are not elementary sentences," and that the tabular sentence-sign of 4.442 "expresses a single truth-function of elementary sentences, even when 'p' and 'q' are truth-functions of elementary sentences." How so? If "p" and "q" are not logically independent sentences, they will not have all four of the truth-possibilities enumerated in the tabular sentence-sign.[6]

The closest the *Tractatus* comes to a discussion of this topic is 5.5, which uses a generalization of the abbreviation of tabular sentence-signs from 4.442 to introduce the generalized N-truth-operation (joint denial), "(————T)(ξ, ...)", saying:

> This operation negates all the sentences inside the brackets on the right, and I call it the negation of these sentences.[7]

I gloss this explanation thusly: the N-operation applied to truth-functions of elementary sentences—an elementary sentence is a truth-function of itself (5b)—yields that sentence which expresses agreement with any truth-possibility of elementary sentences with which each of these truth-functions disagrees and expresses disagreement with any truth-possibility of elementary sentences with which at least one of these truth-functions agrees.[8] This is what it is for an operation to 'negate' all of the sentences to which it is applied; this is what applications of the N-operation all have in common. Similarly, when in a tabular sentence-sign expressing the disjunction of two elementary sentences, we replace elementary sentences with truth-functions of elementary sentences, we get an expression of agreement with any truth-possibilities of elementary sentences with which one or both of the disjuncts agrees and of disagreement with any truth-possibilities of elementary sentences with which both of the disjuncts disagree.

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The *Tractatus* accordingly distinguishes the truth-arguments of a truth-function from the bases for the application of a truth-operation. The truth-arguments are those elementary sentences with whose truth-possibilities a particular truth-function expresses agreement and disagreement. The bases for an application of a truth-operation are those sentences from which the truth-operation, in its distinguishing way, produces a truth-function of the truth-arguments of the bases.

Let us now explore an alternative to the Tractarian account of truth-operations and truth-functions. The *Tractatus* brings the modal aspect of the concept of truth into its characterization of truth-functions by reference to the truth-possibilities of elementary sentences. But why the restriction to elementary sentences? Every sentence as a logical picture of reality represents a possible situation in logical space: it is both possibly true and possibly false. So, every collection of sentences with sense will then have truth-possibilities. It appears then that the disjunction of two sentences with sense can be directly characterized as an expression of agreement and disagreement with the truth-possibilities of its disjuncts without mention of any sentences of which the disjuncts are truth-functions.

We must, however, take some care in how we formulate an alternative to the Tractarian characterization of disjunction. It will not do simply to replace "elementary sentence" with "sentence" in talking about truth-possibilities and say:

> A disjunction of two sentences is an expression of agreement with the three truth-possibilities in which either one or both disjuncts are true and disagreement with the truth-possibility that both are false.

As noted above, if the two disjuncts are not independent of each other, then they do not have the four truth-possibilities of independent sentences. So, if two sentences A and B are independent, their disjunction is an expression of agreement with the three possibilities in which one or both of A and B are true and disagreement with the possibility that both A and B are false. However, if A contradicts B (but is not equivalent to the negation of B), then the disjunction of A and B is an expression of agreement with the two possibilities in which one of A and B is true and the other false and of disagreement with the possibility that both A and B are false. But then disjunction appears to have a different significance in these two cases. The use of tabular notation to express these truth-functions makes the difference vivid: the tabular expressions will have different numbers of rows.

It seems easy to avoid this difficulty: we can generalize over the truth-possibilities of the disjuncts as the Tractarian account generalizes over the truth-possibilities of the elementary sentences that are truth-arguments of the disjuncts and thus obtain:

> A disjunction of two sentences is an expression of agreement with any of the truth-possibilities of the two sentences in which one or both of them are true and disagreement with any truth-possibility in which both sentences are false.

If the two disjuncts are contradictory, there will be no truth-possibility that verifies them both, just as on the Tractarian characterization there will be no

truth-possibility of their truth-arguments that verifies them both. This alternative characterization appears to be on all fours with the Tractarian characterization. What is wrong with it?[9]

Disjunction must have the same significance regardless of the possible situations represented in the sentences disjoined, regardless of the senses of those sentences. Bearing this point in mind, let us consider the conceptions of *truth-possibility* at play in the Tractarian characterization of dyadic disjunction and the suggested alternative.

Every sentence is qua logical picture either true or false, and possibly true and possibly false. These two truth-possibilities are mutually exclusive and jointly exhaustive; they are independent of the particular situation in logical space a sentence represents. *A fortiori,* the same holds for elementary sentences. In addition, elementary sentences are intrinsically independent. Because of the independence of elementary sentences, the mutually excluding and jointly exhausting possibilities of truth and falsity for individual elementary sentences also fix the mutually excluding and exhausting *joint* truth-possibilities of elementary sentences generally, of collections of elementary sentences, and do so independently of the particular states of things whose obtaining is asserted by them. The joint truth-possibilities of a collection of elementary sentences is, so to speak, the product of the individual truth-possibilities of those sentences and so determined by the number of those sentences.[10] In this way, the truth-possibilities of elementary sentences are homogenous. This homogeneity is displayed by the procedure for the tabular enumeration of the truth-possibilities of a list of elementary sentences illustrated in the examples of 4.31. It is, I claim, this homogeneity that licenses generalizing over truth-possibilities of elementary sentences in characterizing *ways* of truth-functionally combining sentences, i.e., individual truth-operations, independently of the sentences combined, of the bases for the operation.

No such homogeneity characterizes the truth-possibilities of sentences generally. Although individual sentences have the excluding and exhausting possibilities of truth and falsity, the individual truth-possibilities of sentences generally do not determine the excluding and exhausting joint truth-possibilities of collections of sentences. These, we saw, depend on the logical relations among the sentences in the collection and so on the possible situations they represent. I suggest that as Wittgenstein viewed matters, the way in which the joint truth-possibilities of a collection of sentences is exclusive and exhaustive is intrinsic to them. The joint truth-possibilities are, if you will, individuated by the way in which each excludes the others and the way in which each together with the others is exhaustive. So, the possibility that two independent sentences are both true is not the same as the possibility that two sentences, the first of which implies the second without being implied by it, are both true. There are then six different concepts of the joint truth-possibilities of two sentences corresponding to the six ways in which two sentences with sense can be logically related. The only conception of the joint truth-possibilities of two sentences generally is a disjunction of these concepts. Hence, any classification of the truth-possibilities of two sentences generally—for example,

truth-possibility in which one or both of the sentences is true—must be defined in terms of its application to the six cases.

The alternative approach faces a further problem. On the alternative approach, any truth-operation applicable to a fixed finite number of bases can be disjunctively characterized. However, generalized truth-operations, like the N-operation, apply to any number of bases, finite or infinite. They figure in the *Tractatus* analysis of first-order quantification in terms of the application of generalized truth-operations to the arbitrarily many bases given by a sentence-function.[11] Since the logical relations that may hold over arbitrarily many sentences cannot be listed, there will not even be a disjunctive characterization of generalized truth-operations.

From Wittgenstein's perspective, this heterogeneity of the joint truth-possibilities of sentences generally shows that the suggested alternative to the Tractarian characterization of dyadic disjunction is spurious.[12] This characterization generalizes over these heterogeneous joint truth-possibilities and applies common classifications to them. When the disjunctive character both of the generalization and the classifications is made explicit, we are left with a case-by-case characterization of dyadic disjunction to pairs of sentences logically related in different ways; a common element that distinguishes dyadic disjunction disappears. The alternative characterization thus fails to bring out what all dyadic disjunctions have in common regardless of the sense of the disjuncts. I find it tempting to bring in formal concepts from the 4.12s to make this point. Here is the idea. The homogeneity of the truth-possibilities of elementary sentences described above makes the concept of the truth-possibilities of elementary sentences a formal concept. This formality gives the idea of a truth-operation its role and place in logic.

So, the alternative strategy of generalizing over the truth-possibilities of sentences generally to characterize truth-operations fails: it does not deliver an account of the univocal significance of a truth-operation from application to application. In contrast to the rejected strategy, Wittgenstein's idea is to characterize individual truth-operations in terms not of the truth-possibilities of sentences generally, but rather in terms of the truth-possibilities of intrinsically independent sentences that 'underlie' sentences generally. "Underlie" in turn is understood in terms of the truth-functional combinability intrinsic to logical pictures. Sentences with sense are all to be truth-functions of intrinsically independent building blocks; to represent a possible situation in logical space is to represent a possibility of the holding and not holding of intrinsically independent states of things (2.201 and 2.202). In this way then, the *Tractatus* understanding of truth-operations motivates the characterization of sentences generally as truth-functions of independent elementary sentences. As Wittgenstein might have thought of it, the univocal significance of truth-operations across their applications exhibits that sentences generally are truth-functions of independent elementary sentences. This motivation fits with 5.5562:

> If we know on purely logical grounds that there must be elementary sentences, then everyone who understands sentences in their unanalyzed form must know it.[13]

Everyone does appreciate from the unanalyzed sentences of colloquial language that disjunction, negation, etc., each has a uniform significance from application to application.

My account of the motivation for the independence of elementary sentences thus develops Anscombe's suggestion that the "application of the theory of truth-functions" requires "a class of mutually independent propositions" in order that the truth of tautologies be "formal truth."[14] My idea has been that the independence of elementary sentence is required for the homogeneity or 'formality' of the exclusive and exhaustive truth-possibilities with which a Tractarian truth-function is an expression of agreement and disagreement. This homogeneity generates the 'formal truth' of tautologies. This account of the motivation for the *Tractatus* identification of the general sentence-form is not a deductive argument that derives its two salient features from independently compelling principles. Rather, I take it to be an articulation of what is implicit in Wittgenstein's general conception of sentences as logical pictures of the facts.

Once sentences generally are characterized as truth-functions of intrinsically independent elementary sentences, then the concept *truth-possibilities of elementary sentences* is available to characterize implication, which is then used to characterize sameness of sense (5.101, 5.11, the 5.12s). We can now appreciate how the metaphor of logical space and the idea of logical form as what all representations of reality share with reality are to be understood. 3.4 says, "A sentence determines a place (Ort) in logical space." 3.411 elaborates: "A geometrical place and a logical place are alike in that both are the possibility of an existence." A geometrical place is where a body can be located, although no particular thing need be there. Similarly, a place in logical space is a possibility that may obtain, but need not. We pick out a geometrical location by means of a coordinate system. Coordinate systems may be partial—may provide specifications for locations restricted to some limited portion of geometrical space, perhaps because a system is tied to the contingent location of some body. We can think of representations with different pictorial forms as using different coordinate systems to specify locations. The logical form that all representations share with reality guarantees that their different coordinate systems all determine positions in logical space *überhaupt* so that the position specified by any representation is related to that specified by any other. (See 3.42.)

Logical form does this by providing a uniform global system of coordinates in which special and limited systems of coordinates are all embedded. The metaphor of logical coordinates for locations in logical space is cashed out in terms of the elementary sentences which are the truth-arguments of the truth-functions into which every sentence is analyzable. Elementary sentences are themselves combinations of names, simple signs with no representationally significant complexity. These names and their possibilities of combination into elementary sentences (the forms of elementary sentences) are the uniform and global coordinates for specifying any location in logical space. The totality of elementary sentences of which every sentence is a truth-function redeems the metaphor of logical space: this is

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what the unity of logical space comes to. The logical form that all pictures of reality share is simply that any picture of reality can be translated into a truth-function of elementary sentences. (See 3.343a and 3.261a.)

II

According to the general sentence-form, every statement with sense is a truth-function of independent elementary sentences. What purchase do we have on this identification? It is analysis that gives the general sentence-form application to the statements of colloquial language, and among these are thoughts themselves (3.5, 4). In the foreword to the *Tractatus,* in summarizing the point of the book, Wittgenstein says, "What that can be said at all can be said clearly," and 4.112 adds, "The goal of philosophy is the logical clarification of thoughts." The full analysis of a sentence rewrites it to make clear what truth-function of which elementary sentences the original sentence is. In some cases, such analysis delivers the desired clarification. In other cases, this clarification may variously involve reflection on the enterprise of analysis.[15] Notoriously, the *Tractatus* offers no substantive characterization of simple objects and states of things to guide analysis. What then does Tractarian analysis amount to? To what standards is it subject?

Russell characterized analysis in epistemic terms. Russell requires that the sentences of science be analyzed as logical constructions within the ramified theory of types on a body of sentences whose atomic expressions designate individuals, properties, relations, and forms with which cognizers are acquainted. Acquaintance here is an epistemically fundamental relation. Acquaintance and the ramified theory of types give us the resources with which to analyze the distinctions in content manifest in colloquial scientific language. Any apparent distinction in scientific discourse that remains uncaptured in these terms is to be rejected as cognitively idle.[16] Russellian acquaintance thus provides a self-standing conception of the *terminus ad quem* of analysis to guide the activity of analysis. It is self-standing in that we identify the sentences at the ground level of analysis in terms of their content or our grasp of their content, and so identify them independently of their role in analysis. In this way, Russellian analysis is bottom-up. We begin with statements about items with which we are acquainted and go on logically to construct from them statements that paraphrase colloquial scientific statements.

In contrast, as Michael Kremer has argued, Tractarian analysis is top-down.[17] I claim that on the *Tractatus* view of analysis, the general sentence-form *by itself* is the complete and sufficient specification of the *terminus ad quem* of analysis. In particular, there is no characterization of the independent elementary sentences apart from the role the general sentence-form gives them as the basis for all construction of sentences from sentences by means of truth-operations.

In order to appreciate the resources for analysis provided by the general sentence-form, we need to consider the forms of elementary sentences. The 2.0s introduce

two mutually dependent notions of form operative at the level of simple objects and states of things. The form of an object is its possibilities for combining with objects to constitute states of things (2.0141). The structure of a state of things is the way in which objects are related to each other in it; the form of a state of things is the possibility that objects be related in this way (2.032; 2.033). I take it that at the bottom level of analysis, there are parallel and isomorphic notions of the form of a Tractarian name and the form of an elementary sentence.

At 5.55, Wittgenstein raises "the question about all possible forms of elementary sentences," but prescinds from any characterization; indeed at 5.5571 he warns against any attempt to do so in advance of analysis. Nevertheless, I venture a suggestion that I believe to be compatible with Wittgenstein's strictures. The form of an elementary sentences is an *unordered* array of argument-places. An argument-place is marked for the form of its arguments; different argument-places for objects of the same form are marked as to whether their arguments must be different objects or whether the same object may occupy both argument-places.[18] The forms of a state of things are individuated by the number and marks of its argument-positions. These unordered arrays of argument-places are the particular ways in which objects can be related to each other to constitute states of things—this is how the objects in a state of things "hang one in another like the links of a chain" (2.03).[19]

This suggestion then rules out that the forms of elementary sentences be the relation-relata forms of atomic formulas of familiar quantificational languages: relation-letters with some fixed finite number of ordered argument-places. It receives some support in succeeding passages in the 5.55s. 5.553 notes that Russell assumes that atomic sentences do have relation-relata forms, and goes on to ask between what numbers of things are there relations. This question inevitably presents itself, if elementary sentences do have relation-relata forms. The 5.554s indicate that something is wrong with the question. 5.5541 says that I am supposed to be able a priori to tell "whether I can get into a position in which I need to signify something with the sign for a 27-termed relation." 5.5542 responds:

> May we then ask this at all? Can we set up a form of sign without knowing whether anything corresponds to it?

5.555b says, "Where we can devise symbols according to a system, this system is the logically important thing and not the individual symbols." This last point is the crucial one.

Tractarian logical resources enable the construction of signs in which the ordering of argument-places is significant and enable the construction, in effect, of signs for relations with intrinsic order-properties. Here are three very simple examples. Suppose

R[x,y][20]

is a form of elementary sentences which takes as arguments different names of some particular form. First, observe that Tractarian logical resources suffice to

construct sentences with three, four, or more argument-places that give the order of the argument-places significance:

$$Ø(x,y,z) =_{df} R[x,y] \,\&\, R[y,z]$$

Note this construction gives significance to the occurrence of the "y" between the "x" and "z."[21] Here then is an example of the order of argument-places in a sign as something that is constructed, something that pertains to the system. We also see here a method for generating signs with an arbitrary number of argument-places. Second, we can also construct a sign for a dyadic relation which one thing can possibly bear to another—a sign whose two argument-places are ordered. Suppose "ψ(x)" is an elementary sentence-function.[22] We can now introduce a sign for a relation:

$$Θ(x,y) =_{df} ψ(x) \,\&\, R[x,y]$$

Note how the definition gives significance to the order of the argument-places in "Θ(x,y)". The relation defined here is neither symmetric nor asymmetric. Third, we can construct a sign for an intrinsically asymmetric relation:

$$Ø(x,y) =_{df} ψ(x) \,\&\, {\sim}ψ(y) \,\&\, R[x,y].$$

As in the second example, this definition gives significance to the order of the argument positions in "Ø(x,y)." The relation thus constructed is intrinsically symmetric in that

$$Ø(x,y) \supset {\sim}Ø(y,x)$$

is a tautology.[23] My idea is this: since signs whose argument-places are ordered can be constructed from elementary sentences whose argument-places are unordered, the order of the argument-places is not fundamental. If order can be constructed, then order must be constructed.[24] Later I shall present a further motivation for this view of the forms of elementary sentences.

We are now ready to consider Tractarian analysis. 5.557 says:

> The *application* of logic decides what elementary sentences there are.
> What lies in its application, logic cannot anticipate.
> This much is clear: logic is not permitted to collide with its application.
> But logic must touch its application.
> Therefore, logic is not permitted to overlap its application.

I take the activity of analysis to be the application of logic. The data for analysis are the manifest logical relations among the sentences with sense of colloquial language. The task for analysis is to rewrite these sentences to make explicit what truth-functions of what elementary sentences each one is. At the level of elementary sentences, the resources for analysis are the interlocking differences among forms of objects and forms of states of things and, within forms of objects, the number of objects of that form. The application of analysis decides what elementary sentences there are in that analysis finds the multiplicity of forms, and within forms of objects the multiplicity of objects, needed in order to capture the logical relationships manifest in colloquial language. I take this to be the point of 5.5563:

> In fact, all the sentences of our colloquial language are in perfect logical order, just as they are.—That utterly simple thing, that we ought to state here, is not a simile of the truth but the complete truth itself.

We should not expect the vocabulary of elementary sentences to be familiar vocabulary—a theme to which I shall return in section 3. Rather, our understanding of elementary sentences is exhausted in our understanding how the identification of colloquial sentences with truth-functions of elementary sentences captures the manifest logical relationships among the colloquial sentences. So, we have no grasp on what different forms of objects are, except via the interlocking contrasts among those forms that give different forms of elementary sentences different roles in capturing manifest logical relationships. The same holds for our grasp on the multiplicity of objects within a form. Our knowledge of the individual objects is exhausted in our appreciation of how just that number of objects of that form contributes to capturing manifest logical relationships. As a result, forms and objects cannot be known individually, but only all together, as features of a system. 5.524 says:

> If objects are given, then with them all objects are given.
> If elementary sentences are given, then with them all elementary sentences are given.

It is then in the sentences of colloquial language, whose sense analysis makes explicit, that we are given all objects and all elementary sentences.

Brian McGuinness nicely encapsulates this view of Tractarian objects:

> Thus we must not think of the realm of reference as a mysterious, infinitely extended magazine of things, as if they were concrete objects, with which we might or might not be lucky enough in a full life to have acquaintance by, so to speak, coming across them in a street. There is already contained in language and thought the possibility of all objects that are possible. All logical forms are logically possible within language, within thought.[25]

This view of Tractarian objects explains why Wittgenstein avoids offering any informative characterization of what kinds of objects there are or might be. Given the holism of *Tractarian* analysis, there is no such characterization in advance of analysis. There is only the rewriting of sentences to make explicit what truth-functions of which elementary sentences they are. These analyzing sentences, in clarifying the possibilities represented in the analyzed sentences by locating them vis-à-vis the others in logical space, elucidate the meanings of the primitive signs, the names, in the analyzing sentences.[26]

It is useful to approach these matters from another direction. 5.526 says:

> We can describe the world completely by means of fully generalized sentences, i.e., without first correlating any name with a particular object.
> Then, in order to arrive at the customary mode of expression, we simply need to add, after an expression 'There is one and only one x such that . . .', the words: and this x is a.

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This remark is puzzling. Tractarian generalizations result from applying a truth-operation to the instances of the generalization that are the values of the corresponding sentence-function. How then can names be dispensable? To understand 5.526, consider 4.26:

> The specification of all true elementary sentences describes the world completely. The world is completely described by the specification of all elementary sentences plus the specification which of them are true and which are false.

Suppose we have a true existential generalization of some truth-function of elementary sentences. We can now introduce names and use them to form true instances of the generalization:

> "(∃x,y)Ø(x,y)" Let a and b be two things such that Ø(a,b).[27]

The true truth-functions obtained in this way will constrain the truth-value of various of their truth-arguments. Elaborations of the generalization and consideration of further generalizations provide a basis for introducing further names, and more tightly constrain the co-occurrences of these names in true elementary sentences. I take 5.526 to maintain that the collection of true fully generalized sentences fixes the truth-value of elementary sentences containing the names introduced on the basis of the generalizations.[28] If this is so, there is at least one way of the names introduced on the basis of coordinating the generalizations and the names appearing in the exhaustive specification of true elementary sentences of 4.26 which yields a 1–1 mapping which associates each true elementary sentence from the specification with a true elementary sentence constructed from the body of true generalizations, and vice versa. The only difference between these two exhaustive specifications of elementary sentences describing the world is a difference in the coordination of name with object. Hence, the body of true generalizations offers a description of the world that contains all the information contained in the exhaustive specification of true elementary sentences. To this extent then, I agree with Hidé Ishiguro's insightful comparison of Tractarian names with the 'dummy names' used in those stretches of mathematical reasoning represented in familiar natural deduction systems by the use of free variables introduced into deductions by rules for universal and existential instantiation.[29]

We can now appreciate the force of 2.0232's vivid metaphor: "Objects are, as it were, colorless." There are no criteria of individuation for objects of the same form, no intrinsic characteristics or kinds of characteristics that distinguish them: the very idea of criteria of individuation makes no sense in application to them.[30] Rather,

> Two objects of the same logical form—apart from their external properties—are only differentiated from one another in that they are different. (2.0233)

The *Tractatus* leaves open the possibility that multiple objects share the same form and that objects of the same form have all their properties in common (2.0233

and 5.5302). 2.02331 accordingly concludes that there is no guarantee that we can distinguish individual objects of the same form:

> For if nothing distinguishes (hervorheben) a thing, then I cannot distinguish it, for otherwise it would be distinguished.

However, analysis does not require us to distinguish the objects. Analysis may reveal the statements of everyday language to be generalizations over simple objects.[31] In this case, analysis requires only a grasp of the number of objects of the interlocking forms necessary for capturing the logical relations manifest in colloquial language.

Wittgenstein thus reworks Russell's and Frege's understanding of the quantificational logic they devise. Russell explicitly and Frege implicitly take their logic to be the logic of relations. Within logic, various kinds of relations can be characterized in logical terms and the theories of those kinds of relation deductively developed. Frege's development of the theory of the ancestral of a one-one relation in chapter 3 of *Begriffsschrift* is the paradigm. This feature of their logical systems figures centrally in Frege's and Russell's efforts to establish logicism.

Wittgenstein turns all this on its head. What is central to representation generally and so to logic are considerations of multiplicity,[32] and these considerations are understood in terms of differing forms and the numbers of objects within the forms of objects. The general sentence-form itself makes no assumptions about the forms or number of objects. The application of logic determines this, and this is all the application of logic determines. The representation of all differences in sense in terms of interlocking multiplicities at the level of objects and elementary sentences is how logic touches, but does not overlap its application. Here is a further motivation for identifying the forms of elementary sentences with unordered arrays of argument-places. It allows form and content to be captured at the bottom level of analysis entirely in terms of multiplicities, and in doing so, marks a clear boundary between logic and its application.

III

I have characterized Tractarian analysis in the most abstract terms. It thus remains opaque how the application of logic might capture the logical relations manifest in colloquial language. We make progress in analysis, when we paraphrase colloquial sentences into familiar quantificational notation, guided by the goal of maximizing the implications over our target group of sentences representable in quantificational terms. Moreover, a judicious selection of simple predicates for use in our paraphrases, will enable us to define logically a host of other predicates in terms of our chosen primitives and thus to exhibit the quantificational character of yet more implications. Nevertheless, in this enterprise, we quickly run up against implications we cannot capture in purely quantificational terms. These apparent material necessities pose an ostensibly insuperable obstacle to further progress in Tractarian analysis.

In this section, I shall present two strategies for accommodating material necessities. The availability of these strategies gives some reason for thinking that Tractarian analysis can accommodate apparent material necessities. I present my two strategies by outlining an approach to the Tractarian analysis of statements describing a simple model-world. My discussion has the added dividend of providing some idea of what Tractarian simple objects might be like.

My model-world is a line divided into discrete segments and infinitely extended in both directions. Each segment is uniformly colored. We can describe the colors of finite segments of the line:

> There is a black segment followed on one side by gold, blue, and green segments respectively, and on the other by green, pink, and purple segments respectively.

We can describe distributions of colors over the line:

> Every second segment is scarlet.

As far as these descriptions go, the possibilities latent in the model-world are the possibilities of color for each segment: any segment can have any color, and have any color independently of the colors of any other segments.

Frege's quantificational paraphrases of statements of number illustrates one way of capturing apparent material necessities in quantificational terms. Statements of number are statements that answer the question "How many?"—for example, "There are exactly two Martian moons." Our example goes over into

> $(\exists x)(\exists y)[x \text{ is a Martian moon } \& \ y \text{ is a Martian moon } \& \ x \neq y \ \&$
> $(\forall z)(z \text{ is a Martian moon } \supset (z = x \vee z = y))]$.

Note how it follows from this statement that there is not only one Martian moon, nor three, four, or more.

I take the brief discussion of statements "expressing the degree of a quality" in Wittgenstein's 1929 paper "Some Remarks on Logical Form" to indicate one way in which he applied this paradigm in Tractarian analysis:

> One might think—and I thought so not long ago—that a statement expressing the degree of a quality could be analyzed into a logical product of simple statements of quantity and a supplementing statement. As I could describe the contents of my pocket by saying "It contains a penny, a shilling, two keys, and nothing else." This "and nothing else." is the supplementary statement which completes the description.[33]

Wittgenstein goes on to consider statements ascribing degrees of brightness to possibly luminous objects. Suppose there is some fixed number of degrees of brightness, say five.[34] We now introduce unfamiliar objects, one for each of the degrees of brightness—call them brightness units—and a relation *HAS* between possibly luminous objects and brightness-units.[35] A possibly luminous object may have none, some, or all of the brightness units. The sentence "A has exactly three degrees of brightness" can now be paraphrased by the familiar quantificational paraphrase

of "There are exactly three different brightness units x such that A HAS x." What does this existential generalization come to in Tractarian terms?

Here I need to interpolate a brief discussion of the *Tractatus* understanding of identity and quantifiers. First, 5.53 tells us that identity of object is expressed by identity of sign and difference of object by difference of sign, and not by the dyadic identity-predicate. Second, the *Tractatus* understands a universal (existential) generalization to be the result of applying the truth-operation of generalized conjunction (disjunction) to the instances of the generalization—in Tractarian terms to the sentences that are values of the sentence-function that in Russell-Peano notation occupies the scope of the quantifier. Third, in generalizations identity and difference of quantified variables (as we call them) also express identity and difference of object, when the scopes of the governing quantifiers are embedded. So, "(∀x)(∃y)Fxy" says that everything bears F to something else; it is then a conjunction of disjunctions:

(Fab ∨ Fac ∨ . . .) & (Fba ∨ Fbc ∨ . . .) & (Fca ∨ Fcb ∨ . . .) & . . .

Similarly, "(∃x)Fax" says that a F's something else. Finally, on its Tractarian construal "(∀x)(∃!y)Fxy" ("Everything F's exactly one other thing") is the truth-function:

{[Fab & ~Fac & ~Fad & . . .] ∨ [Fac & ~Fab & ~Fad & . . .] ∨ . . .]} &
{[Fba & ~Fbc & ~Fbd & . . .] ∨ [Fbc & ~Fba & ~Fbd & . . .] ∨ . . .} & . . .[36]

So assuming that there are five possible degrees of brightness, on the suggested analysis, "A has three degrees of brightness" is a disjunction that runs through the ten ways that A can have exactly three of the five brightness-units. Wittgenstein appears to have thought that some complication of this strategy for handling attributions of brightness degrees could similarly handle color-attributions,[37] and I will follow him here as regards the analysis of attributions of colors to line segments in descriptions of the model-world.

To explore a second strategy for handling material necessities, let us consider how invocation of the structure of the segmented line is to be handled in Tractarian analyses of descriptions of the model-world. Consider the relation of adjacency over our line segments. Here is a relation that takes as arguments two different line segments. Adjacency has an essential structure, an intrinsic multiplicity, that can be characterized in quantitative terms:

> Every segment stands to every other in the ancestral of the adjacency relation. Moreover, every segment is adjacent to just two others, and each of these, being adjacent to the first segment, is adjacent to exactly one more segment ("on the other side," so to speak); each of these two further segments is adjacent to exactly one further segment, and these two new segments in turn . . . .[38]

I shall refer to this characterization of the structure of the line by "M", "M" for "multiplicity condition." M presents itself as an intrinsic characterization of adjacency, and so an ostensible material necessity that bounds the possibilities latent in the model-world. I see no prospect of *defining* adjacency in terms of independent

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elementary sentences in a way that will make M a tautology. My second strategy explains away the apparent necessity of M by claiming that M is generally presupposed by descriptions of the model-world.

To understand the second strategy, consider 3.24. 3.23 says, "The requirement that simple signs be possible is the requirement that sense be determinate." Following on it, 3.24 says:

> A sentence concerning a complex stands in an internal relation to sentences concerning its components.
>
> The complex can only be given by means of its description, which will either be correct or incorrect. A sentence that speaks of a complex will be simply false, not nonsense, if the complex does not exist.
>
> That an element in a sentence designates a complex can be seen in an indeterminacy in the sentences in which this element occurs. We *know* by means of this sentence that not everything is determined (Indeed, the generality-designation *contains* a prototype (Urbild).)
>
> The combination of the symbols for a complex into a simple symbol can be expressed by means of a definition.

It is important not to be trapped in an overly superficial understanding of the idea of a complex at play here. Let me rehearse what I warn against. 3.24 might suggest that analysis in significant measure proceeds along the following lines. Given a body of colloquial sentences, search out mention of complexes. These will be the things whose existence is a factual matter, whose existence must be a matter of other things standing in various relations. Of course, the things whose standing in these complex-constituting relations may themselves be contingently existing complexes. This analytic maneuver will have to be repeated until we reach things whose existence is not contingent. The problem is that analysis guided by this 'knee bone connected to the thigh bone' conception of complexity leads us only from things that break apart into salient units to contingently existing things that don't. It brings us no closer to things with the marks specified for simple objects in the 2.02s.

It may look as if the line segments, together with the items like brightness-units to represent colors, constitute the simple objects of the model-world. Brief reflection shows that the segments themselves are manifestly not Tractarian simple objects. From the perspective of colloquial language, the possibilities latent in the model-world are possibilities of distributions of colors over the line segments. Suppose there is a black segment next to a gold one. It is possible that the colors of the two segments be reversed, that the black segment be gold and the gold one black. But it makes no sense to say that the segments themselves change positions. Rather, the segments are individuated by their relative positions on the line. They *are* the positions.

I propose to take the segmented line of the model-world to be a paradigm of a Tractarian complex. The components of this complex are *not* the line segments, for these are individuated only by reference to the complex. Rather, the line is composed of underlying simple spatial units, analogous to the brightness-units. There is a single way of combining any two different spatial units into a state of things.

The existence of the line is a matter of these states of things satisfying condition M. We need then to consider how M can be expressed as a truth-function of elementary sentences.

Truth-functions are formed by applying a truth-operation to a collection of sentences, the bases for the truth-operation. 5.501 describes three ways of specifying the bases for the application of a truth-operation. First, there is direct enumeration: specifying the bases of a truth-operation by listing them. The second means is by use of a sentence-function—the means employed in universal and existential generalizations. The third way of specifying the bases is to give a formal law for constructing a series of sentences; the bases of the truth-operation are the members of the series. What Wittgenstein has in mind by a formal law is some kind of notationally significant recursive procedure for generating a series of sentences, what the *Tractatus* calls a form-series (Formenreihe).[39]

The only example of a form-series in the *Tractatus* is given informally at 4.1251 and 4.1273 to show how, in contrast to Frege and Russell, the ancestral of a relation is to be defined:

> If we want to express in begriffsschrift the general sentence, "b is a successor of a," then we require an expression for the general term of the form-series:
>
> aRb, (∃x):aRx • xRb, (∃x,y):aRx • xRy • yRb, ...
>
> The general term of a form-series can be expressed only by means of a variable, for the concept: member of this form-series, is a *formal* concept. (Frege and Russell overlooked this. The way they want to express general sentences like the above is therefore incorrect; it contains a circulus vitiosus.)
>
> We can determine the general series of forms by giving its first term and the general form of the operation that produces the next term out of the sentence that precedes it.

The sentence "a is an R-ancestor of b" is the result of applying the truth-operation of generalized disjunction to the sentences of this form-series.

By use of form-series, condition M, applied to the way in which any two different spatial units can be related to constitute a state of things, can be expressed as a Tractarian truth-function. I use "R[x,y]" to signify the form of elementary sentences asserting the obtaining of these states of things. The existence of the line is a matter of these states of things satisfying condition M. Let M(R) be the sentence saying that these states of things satisfy condition M. This sentence, M(R), is the description of the complex. We had taken the segments to be related to the color-constituting units. We now let the spatial units play this role.

Colloquial descriptions of the model-world will be analyzed in terms of generalizations over the spatial units, just as attributions of degrees of brightness were analyzed by generalizations over brightness units. Furthermore, on account of their mention of line segments, the colloquial descriptions presuppose the existence of the line. This presupposition is represented by analyzing every colloquial description of the model-world as a conjunction one of whose conjuncts is M(R).

Consider now the colloquial sentence, "There is a black segment next to a gold segment." This statement is analyzed by:

$$M(R) \,\&\, (\exists x)(\exists y)(R[x,y] \,\&\, \Psi(x,y)),$$

where "$\Psi(x,y)$" abbreviates the color attributions. M(R) may hold or not. If it does not hold, then the colloquial sentence is false. What about the grammatical negation of our sample description, "No black segment is next to a gold one"? Before analysis, we view this sentence as *the* negation of the original, but in analyzing it, we recognize it to be:

$$M(R) \,\&\, \sim(\exists x)(\exists y)(R[x,y] \,\&\, \Psi(x,y)).$$

Thus, our grasp of the colloquial sentence was cloudy, because we elided the distinction expressed after analysis by the different scopes given to the negation sign. The failure to distinguish these two negations is an indeterminacy in the colloquial sentence.[40] Because colloquial descriptions of the model-world go over into conjunctions, one of whose conjuncts is M(R), the relative locations in logical space determined by these sentences all lie within the region demarcated by M(R).

The line appears to have an intrinsic structure that is reflected in the apparent material necessity of the colloquial statement

(1) (The relation of) Adjacency (over line segments) satisfies M.

As this is a colloquial description, it presupposes the existence of the line and so is analyzed as a conjunction that has M(R) as a conjunct. Indeed, the analysis of (1) just is the conjunction:

$$M(R) \,\&\, M(R),$$

which of course is just M(R). That every colloquial sentence (and its colloquial negation) has the analysans of (1) as a conjunct is what gives this statement its appearance of necessity. This appearance is reinforced when we reflect that the colloquial negation of (1)

(2) Adjacency does not satisfy M

is analyzed by

$$M(R) \,\&\, \sim M(R),$$

which is a genuine contradiction. Sensing the contradictory character of (2), we take (1) to be necessarily true.

Here then is a second way the *Tractatus* has to handle apparently material necessities. Apparent necessities may arise from the designation of a complex in an incompletely analyzed sentence. That is, the apparent necessity may be presuppositional, and hence shown by analysis to be no necessity at all, despite appearances. The presupposition, the mention of the complex may be tacit, as in the case of descriptions of the model-world. These descriptions need not explicitly mention the line. They are about ostensible line segments. It is because the line segments are individuated

in terms of the structure of the complex that constitutes the line that a description of this complex is a part of the sense of colloquial descriptions of the model-world.

I think that in the hard cases, the important cases, apparent material necessities concern the structure of a complex. Tractarian analysis requires that at the ground level of analysis any structure whose existence is presupposed at the penultimate level be fully described in terms of truth-functions of independent elementary sentences. The apparently necessary structure is generated by quantitative features of the underlying states of things that are set forth in the ground-level description of the complex. This strategy for explaining away apparent material necessities is most persuasive when it uncovers a complex whose structure is tacitly presupposed by a large and central body of statements. The availability of this strategy should assuage someone who worries that Tractarian resources are too meager to do justice to the 'structural' material necessities, like those of geometry, pervading colloquial descriptions of reality, especially someone who finds the top-down conception of analysis otherwise attractive. Of course, this strategy for explaining away apparent necessities will not persuade anyone who is convinced that these necessities are indeed genuine.[41]

I hope it is clear from my discussion of the model-world that Tractarian logical resources applied to countably many objects and a way of combining any two of these objects into states of things suffices to describe a great variety of possible structures. There are limits to this approach to analysis. In particular, Tractarian resources applied to countably many objects will not suffice for a description of the structure of the full continuum. I suspect that Wittgenstein recognized this limit, and was not at the time fazed by it.

Early in this section, I maintained Wittgenstein's description of his earlier view of statements concerning the degree of a quality in "Some Remarks on Logical Form" gives us an example of how he conceived of Tractarian analysis. Wittgenstein describes his view there only to reject it:

> . . . if on the other hand we try to distinguish between the units and consequently write E(2b) = E(b′) & E(b″), we assume two different units of brightness; and then if an entity possesses one unit, the question could arise, which of the two—b′ or b″—it is; which is obviously absurd.[42]

Why might Wittgenstein find this view absurd? The statement that a dimly glowing thing has exactly one degree of brightness is analyzed by the statement that says that the thing possesses exactly one brightness-unit. If this analysis is correct, then it must be intelligible to ask which of the brightness-units the thing has. We understand what it is for a thing to possess just one degree of brightness and we understand what it is to be able to discriminate among five degrees of brightness. But we have no understanding of what distinguishes the individual brightness-units or for what it is for a thing to possess one rather than another brightness-unit. So the question that must be intelligible is not.

This is a surprising objection to the *Tractatus.* As noted in §2, apart from their external properties, Tractarian objects of the same form are simply indistinguishably

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different—they have no criteria of individuation. This is the view that Wittgenstein rejects in 1929. More generally, the objection appears to be premised on the rejection of top-down analysis guided solely by the general sentence-form. In the second paragraph of "Some Remarks on Logical Form," Wittgenstein says that it is the job of theory of knowledge to "inspect the phenomena we want to describe" in order to determine the forms of elementary sentences. It is in the context of this new, phenomenally oriented view of analysis that the objection to brightness-units is persuasive. I take its appearance in "Some Remarks on Logical Form" to show that by 1929 Wittgenstein had already turned his back on the *Tractatus* understanding of the general sentence-form.[43] We are left with the question of why he did so.

APPENDIX: ORDER FROM QUANTITY

In order to illustrate how form-series enable order to be extracted from quantity, I show how to define a three-place betweenness-relation from the way in which two different spatial units can combine to form a state of things. Here I use "Rxy" to signify the form of elementary sentences asserting the obtaining of these states of things. The betweenness relation I define will have the desired order-properties, provided that these states of things satisfy condition (M). Condition (M) itself is a conjunction of two truth-functions whose construction relies on form-series. Consider first the disjunction of the sentences of the following form-series:

Rab
$(\exists z_1)(Raz_1 \,\&\, Rz_1b)$
$(\exists z_1)(\exists z_2)(Raz_1 \,\&\, Rz_1z_2 \,\&\, Rz_2b)$
. . .

This disjunction states that a bears the ancestral of R to b. Now convert "a" and "b" into variable-names, and apply the truth-operation of conjunction to the resulting sentence-function. To obtain M(R), conjoin this sentence with the conjunction of the sentences of the following form-series:

$(\forall x)(\exists y_1)(\exists y_2)(\forall z)[Rxy_1 \,\&\, Rxy_2 \,\&\, \sim Rxz]$
$(\forall x)(\exists y_1)(\exists y_2)(\exists y_3)(\exists y_4)(\forall z)[Rxy_1 \,\&\, Rxy_2 \,\&\, Ry_1y_3 \,\&\, Ry_2y_4 \,\&\, \sim Ry_1z \,\&\, \sim Ry_2z)]$
$(\forall x)(\exists y_1)(\exists y_2)(\exists y_3)(\exists y_4)(\exists y_5)(\exists y_6)(\forall z)[Rxy_1 \,\&\, Rxy_2 \,\&\, Ry_1y_3 \,\&\, Ry_2y_4 \,\&\, Ry_3y_5 \,\&\, Ry_4y_6 \,\&\, \sim Ry_3z \,\&\, \sim Ry_4z]$
. . .[44]

The distance between two spatial units is the number of units separating them. Betweenness is easily defined in terms of distance: unit x is between units y and z if the distance separating y and z is greater than the distance separating x from y and the distance separating x from z. The problem is to define distance in

terms of the underlying states of things. It is easy for each N to define a predicate "D(y, z) = N" which says that the distance from y to z is N. For example,

$$D(y,z) = 3 =_{df} (\Sigma v,w,u)(Ryv \,\&\, Rvw \,\&\, Rwu \,\&\, Ruz).$$

Note, if we are given that the distance between y and z is, say 3, we can enumerate necessary and sufficient conditions for x to be between y and z: either the distance separating x and y is 0 and separating y and z is 2, or the distance separating x and y is 1 and that separating y and z is 1, or the distance separating x and y is 2 and that separating y and z is 0. Note, however, the form of the defined predicate is "Ø(x,y)." The "= 3" corresponds to no filled argument-place in the definiens; it is merely a mnemonic device. This is the same limitation that led Frege to reject his regimen of quantificational paraphrase for statements of number as a definition of the concept of number, and motivated him to introduce numbers as logical objects.[45]

Wittgenstein uses form-series to overcome this limitation. By use of a form-series to specify the bases for an application of generalized disjunction, we can by brute force define "a is between b and c" in terms of defined predicates of the form "D(x,y) = N." We collect together in a form-series the infinitely many instances of the schema

$$D(b,c) = N \,\&\, [(D(a,b) = 0 \,\&\, D(a,c) = N\text{-}1) \text{ ß } \ldots \text{ ß } (D(a,b) = N\text{-}1 \,\&\, D(a,c) = 0].$$

Eliminating the distance-sentences in terms of their definiens makes visible the iterated operation in building the series:

$$(\exists x)[(Rbx \,\&\, Rxc)] \,\&\, Rab \,\&\, Rac;$$
$$(\exists x)(\acute{}\exists y)[Rbx \,\&\, Rxy \,\&\, Ryc] \,\&\, (\exists x)\{[Rba \,\&\, Rax \,\&\, Rxc] \text{ ß } [Rbx \,\&\, Rba) \,\&\, Rac]\};$$
$$\ldots$$

"a is between b and c" is defined as the generalized disjunction of the sentences of this form-series. So defined, betweenness has intrinsic order, given M(R). That is, the instances of

$$M(R) \supset (x \text{ is between } y \text{ and } z \,\partial\, \sim(y \text{ is between } x \text{ and } z))$$

are tautologies.

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than three decades ago about simple objects as well as more recent conversations. I am also grateful for a conversation in which Tyke Nuñez spoke what became for me das erlösende Wort. Comments from Michael Friedman, Ned Hall, Peter Hylton, Penelope Maddy, Brian Rogers, Joachim Schulte, David Stern, and Kai Wehmeier improved this paper. I presented this paper at the Third Annual Auburn University Philosophy Conference, at a workshop at the Logic and Philosophy of Science Department at the University of California, Irvine, at the University of Chicago Wittgenstein Seminar, at a workshop at the Universität Leipzig, and at a Wittgenstein workshop at the Universität Zürich. Revisions of the paper benefited from comments I received on these occasions.

NOTES

1. G. E. M. Anscombe, *An Introduction to Wittgenstein's Tractatus* (Philadelaphia: University of Pennsylvania Press, 1959), 25f.
2. Although there is more to be said here, especially in connection with the discussion of the simplicity of Tractarian objects in the 2.02s and at 3.24, I will not address this issue in this paper.
3. Of course, like us, for Wittgenstein, the replacement of any component sentence of a truth-functionally compound sentence by another sentence with the same truth-value does not change the truth-value of the compound sentence. (In saying this, I put to one side ways in which Wittgenstein's view of the structure of truth-functionally compound sentences differs from our view.) My point is that whereas our *characterization* of conjunction, negation, etc., is extensional, Wittgenstein's characterization is intentional.
4. 2.171 gives as an example of pictorial form spatial arrangements of items that represent the relative spatial positions of bodies, like the famous example of a spatial model of the relative positions of cars at an accident scene that Wittgenstein presents in the 29.9.14 *Notebook* entry that first compares sentences to pictures. Such spatial pictures are not independent of each other: the truth of any representation of the positions of the cars at the accident scene rules out the truth of any of the others.
5. Remember, in this remark "truth-function" means "truth-function of elementary sentences."
6. I elaborate this point below.
7. This is the only example of a characterization of an individual truth-operation in the *Tractatus.*
8. So, when we apply the tabular notation to non-elementary sentences, the array of Ts and Fs underneath those sentences generalize over the truth-possibilities of elementary sentences. For example, in the tabular expression of the disjunction of two non-elementary sentences, the first line
   T T T
   indicates that the tabular sentence agrees with any truth-possibility of elementary sentences that verifies both of the bases.
9. I am indebted to Pen Maddy for pointing out the importance of this alternative to the Tractarian characterization.
10. 4.27 gives one mathematical expression of this.
11. For further discussion, see my "Logical Segmentation and Generality in Wittgenstein's *Tractatus,*" in *Wittgenstein's* Tractatus*: History and Interpretation,* ed. Peter Sullivan and Michael Potter (Oxford: Oxford University Press, 2013), 125–42.
12. That Wittgenstein viewed matters in something like this way is suggested by his discussion in the 1929 MS 106 of truth-functional connectives applied to atomic sentences each of whose truth excludes the truth of the others. There he says that dyadic conjunction and disjunction in application to these sentences has a different meaning (eine andere Bedeutung) in application to mutually excluding atomic sentences on the one hand and independent ones on the other. See MS 106,

pp. 87–92. At the end of "Remarks on Logical Form," Wittgenstein says that in the sense in which there is a conjunction of two independent atomic sentences characterized in terms of their joint truth-possibilities, there is no conjunction of mutually excluding elementary sentences.

13. See also 4.221 and 5.555. In 5.555 the conception of elementary sentences is linked to the systematic construction of sentences.
14. *An Introduction to Wittgenstein's* Tractatus, 33.
15. Analysis, as just characterized, does not exhaust the activity of clarifying thoughts. For a discussion of another sort of clarification, see Michael Kremer, "Mathematics and Meaning in the *Tractatus*," *Philosophical Investigations* 25 (2002): 272–303. For a general discussion of the *Tractatus* conception of logical clarification, see Cora Diamond, "The *Tractatus* and the Limits of Sense," in *The Oxford Handbook of Wittgenstein,* ed. M. McGinn and O. Kuusela (Oxford: Oxford University Press, 2011), 240–75.
16. In this sketch, I oversimplify Russell's changing view of analysis.
17. I am indebted to Michael Kremer's discussion of the emergence of a top-down conception of analysis in Wittgenstein's thinking in "Contextualism and Holism in the Early Wittgenstein: From *Prototractatus* to *Tractatus*," *Philosophical Topics* 25 (1997): 87–120. See especially pp. 96–99. Kremer also maintains that Tractarian analysis is a matter of explicitly representing the logical relationships among sentences in terms of truth-functions of independent elementary sentences. The account I give of simple objects generally meshes with Kremer's discussion of Tractarian names on pp. 112–14.
18. I take this last point to be indicated in *Prototractatus* 5.422.
19. Michael Kremer observed to me that my view of elementary sentences resembles Anscombe's view in *An Introduction to Wittgenstein's* Tractatus, as modified in her note "Retractation," *Analysis* 26 (1965): 109–11.
20. I use square brackets to indicate the argument-places are unordered. As a result, "R[a,b]" and "R[b,a]" are typographical variants of the same sign.
21. But not to the relative position of the "x" and the "z": just as "R[x,y] & R[y,z]" and "R[y,z] & R[x,y]" are typographical variants of the same sign, so are "Ø(x,y,z)" and "Ø(z,y,x)."
22. An elementary sentence-function is the result of transforming a name in an elementary sentence into a variable-name. See 4.24. For a discussion of sentence-functions, see section 2 of my "Logical Segmentation and Generality in Wittgenstein's *Tractatus*."
23. The appendix presents a more interesting example: the definition of a triadic betweenness-relation from the two-place form of states of things underlying the adjacency relation in the model-world discussed in §3.
24. I think that 5.551 gives this view some support.
25. Brian McGuinness, "The Supposed Realism of the *Tractatus*," in Brian McGuinness, *Approaches to Wittgenstein: Collected Papers* (London: Routledge, 2002), 91.
26. This is how I understand the characterization of elucidations of primitive signs at 3.263.
27. Bear in mind here Wittgenstein's use of identity and difference of variable-names to express identity and difference of objects of the same form.
28. I include here generalizations whose construction involves form-series. I do not think that the claim 5.526 makes is obvious, and much more would have to be said to make it plausible. I present it here as an interpretation of what Wittgenstein thought.
29. See Hidé Ishiguro, "Use and Reference of Names," in *Studies in the Philosophy of Wittgenstein,* ed. Peter Winch (London, Routledge, 1969), esp. 46.
30. Warren Goldfarb emphasizes this point in "Das Überwinden: Anti-Metaphysical Readings of the *Tractatus,*" in *Beyond the* Tractatus *Wars: The New Wittgenstein Debate,* ed. Rupert Read and Matthew Lavery (New York: Routledge, 2011), 10–11.
31. The discussion of the analysis of sentences mentioning complexes in the 2.02s and 3.2s, *NB* 18.6.15[3] and 21.6.15[12], and the passages in "Some Remarks on Logical Form" to be discussed in §3 all suggest that Wittgenstein anticipated that colloquial sentences would be analyzed as generalizations.
32. See 4.04–4.041.

33. "Some Remarks on Logical Form," *Aristotelian Society Supplementary Volume* 9 (1929): 167.
34. To fix ideas, suppose that there are exactly five just noticeable differences in brightness.
35. If we are at the final level of analysis, HAVING is a way in which a possibly luminous object combines with a brightness-unit to constitute a state of things.
36. For a thorough and definitive discussion that relates the *Tractatus* treatment of quantification and identity to first-order logic with identity, see Brian Rogers and Kai F. Wehmeier, "Tractarian First-Order Logic: Identity and the N-Operator," *Review of Symbolic Logic* 5 (2012): 538–73. I agree with Rogers and Wehmeier that their weakly exclusive interpretation is the view best attributed to the *Tractatus*.
37. Presumably, color-attributions will involve variation along multiple independent dimensions, each dimension being represented by a type of the underlying units. So much is indicated by the discussion of color in §100 of *The Big Typescript, TS 213,* ed. and trans. C. G. Luckhardt and M. A. E. Aue (Malden: Basil Blackwell Publishing, 2005).
38. I am indebted here and in the formalization in the appendix to Ned Hall, Brian Rogers, and Kai Wehmeier for saving me from assorted blunders.
39. See 5.23 and 5.2522. The recursive specification of a form-series may require a logically arbitrary alphabetic ordering of signs. I take this to be permitted as long as any alphabetic ordering of these signs could be employed to specify the form-series. *Prototractatus* 5.005341 says that in the use of form-series to specify the bases for a truth-operation we have a kind of generality, formal generality, which, in contrast the generality of sentence-functions, "was overlooked by Frege and Russell." For further discussion, see my "Logical Segmentation and Generality," §3.
40. The third paragraph of 3.24 concerns this failure to notationally mark scope distinctions.
41. I am indebted to James Shaw and Anil Gupta for urging on me the importance of this point.
42. Wittgenstein, "Some Remarks on Logical Form," *Aristotelian Society Supplementary Volume* 9 (1929): 167f.
43. I am indebted in the last two paragraph to comments from David Stern on this topic.
44. Bear in mind the use of identity and difference of variables to express identity and difference of objects.
45. See Gottlob Frege, *The Foundations of Arithmetic,* §56, p. 68.